**System Calculations**

**Leadscrew Drives**

Leadscrews convert rotary motion to linear motion and come in a wide variety of configurations. Screws are available with different lengths, diameters, and thread pitches. Nuts range from the simple plastic variety to precision ground versions with recirculating ball bearings that can achieve very high accuracy.

The combination of microstepping and a quality leadscrew provides exceptional positioning resolution for many applications. A typical 10-pitch (10 threads per inch) screw attached to a 25,000 step/rev. motor provides a linear resolution of 0.000004" (4 millionths, or approximately 0.1 micron) per step.

A flexible coupling should be used between the leadscrew and the motor to provide some damping. The coupling will also prevent excessive motor bearing loading due to any misalignment.

**Microscope Positioning**

**Application Type:** X/Y Point to Point  
**Motion:** Linear  
**Description:** A medical research lab needs to automate their visual inspection process. Each specimen has an origin imprinted on the slide with all other positions referenced from that point. The system uses a PC-AT Bus computer to reduce data input from the operator, and determines the next data point based on previous readings. Each data point must be accurate to within 0.1 microns.

**Machine Objectives**

- Sub-micron positioning  
- Specimen to remain still during inspection  
- Low-speed smoothness (delicate equipment)  
- Use PC-AT Bus computer

**Motion Control Requirements**

- High resolution, linear encoders  
- Stepper (zero speed stability)  
- Microstepping  
- PC-AT Bus controller

**Compumotor Solution:** Microstepping motors and drives, in conjunction with a precision ground 40 pitch leadscrew table, provide a means of sub-micron positioning with zero speed stability. Conventional mechanics cannot provide 0.1 micron accuracies without high grade linear encoders. It is necessary for the Compumotor Model AT6400 indexer, which resides directly on the computer bus, to provide full X, Y, Z microscope control and accept incremental encoder feedback.
Precision Grinder
A bearing manufacturer is replacing some equipment that finishes bearing races. The old equipment had a two-stage grinding arrangement where one motor and gearbox provided a rough cut and a second motor with a higher ratio gearbox performed the finishing cut. The designer would like to simplify the mechanisms and eliminate one motor. He wants to use a single leadscrew and exploit the wide speed range available with microstepping to perform both cuts. This will be accomplished by moving a cutting tool mounted on the end of the leadscrew into the workpiece at two velocities: an initial velocity for the rough cut and a much reduced final velocity for the finish cut.

The torque required to accelerate the load and overcome the inertia of the load and the rotational inertia of the leadscrew is determined to be 120 oz-in. The torque necessary to overcome friction is measured with a torque wrench and found to be 40 oz-in. A microstepping motor with 290 oz-in of torque is selected and provides adequate torque margin.

This grinder is controlled by a programmable controller (PC) and the environment requires that the electronics withstand a 60°C environment. An indexer will provide the necessary velocities and accelerations. The speed change in the middle of the grinding operation will be signaled to the PC with a limit switch, and the PC will in turn program the new velocity into the indexer. Additionally, the indexer Stall Detect feature will be used in conjunction with an optical encoder mounted on the back of the motor to alert the PC if the mechanics become “stuck.”

Other Leadscrew Drive Applications
- XY Plotters
- Facsimile transmission
- Tool bit positioning
- Cut-to-length machinery
- Back gauging
- Microscope drives
- Coil winders
- Slides
- Pick-and-Place machines
- Articulated arms

Leadscrew Application Data
Inertia of Leadscrews per Inch

<table>
<thead>
<tr>
<th>Diameter In.</th>
<th>Steel oz-in²</th>
<th>Brass oz-in²</th>
<th>Alum. oz-in²</th>
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<tbody>
<tr>
<td>0.25</td>
<td>0.0017</td>
<td>0.0018</td>
<td>0.0006</td>
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<tr>
<td>0.50</td>
<td>0.0275</td>
<td>0.0295</td>
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<tr>
<td>0.75</td>
<td>0.1392</td>
<td>0.1491</td>
<td>0.0478</td>
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<td>1.00</td>
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<td>1.50</td>
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<td>4.1251</td>
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<tr>
<td>2.50</td>
<td>17.1807</td>
<td>18.4079</td>
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<table>
<thead>
<tr>
<th>Diameter In.</th>
<th>Steel oz-in²</th>
<th>Brass oz-in²</th>
<th>Alum. oz-in²</th>
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Coefficients of Static Friction Materials

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<tr>
<th>Material Combination</th>
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<tr>
<td>Steel on Steel</td>
<td>0.58</td>
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<tr>
<td>Steel on Steel (lubricated)</td>
<td>0.15</td>
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<tr>
<td>Aluminum on Steel</td>
<td>0.45</td>
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<tr>
<td>Copper on Steel</td>
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<tr>
<td>Brass on Steel</td>
<td>0.19</td>
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<tr>
<td>PTFE on Steel</td>
<td>0.04</td>
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Leadscrew Efficiencies

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<th>Type</th>
<th>Efficiency (%)</th>
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<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Ball-nut</td>
<td>95</td>
</tr>
<tr>
<td>Acme with metal nut*</td>
<td>55</td>
</tr>
<tr>
<td>Acme with plastic nut</td>
<td>85</td>
</tr>
</tbody>
</table>

* Since metallic nuts usually require a viscous lubricant, the coefficient of friction is both speed and temperature dependent.
System Calculations

Leadscrew Drives

Vertical or Horizontal Application:

- ST – Screw type, ball or acme
- e – Efficiency of screw
- \( \mu_s \) – Friction coefficient
- L – Length of screw
- D – Diameter of screw
- p – Pitch
- W – Weight of load
- F – Breakaway force
- Directly coupled to the motor?
- ST = ________________
- e = ________________ %
- \( \mu_s = ________________ \)
- L = ________________ inches
- D = ________________ inches
- p = ________________ threads/inch
- W = ________________ lbs.
- F = ________________ ounces
- Yes/No

Leadscrew Formulas

The torque required to drive load W using a leadscrew with pitch (p) and efficiency (e) has the following components:

\[
T_{\text{Total}} = T_{\text{Friction}} + T_{\text{Acceleration}}
\]

\[
T_{\text{Friction}} = \frac{F}{2\pi p e}
\]

Where:
- \( F \) = frictional force in ounces
- \( p \) = pitch in revs/in
- \( e \) = leadscrew efficiency

\[
T_{\text{Acceleration}} = \frac{1}{g} \left( J_{\text{Load}} + J_{\text{Leadscrew}} + J_{\text{Motor}} \right) \frac{\omega t}{\omega}
\]

\[
\omega = 2\pi v t
\]

\[
J_{\text{Load}} = \frac{W}{2\pi p} \quad J_{\text{Leadscrew}} = \frac{\pi L p R^4}{2}
\]

Where:
- \( T \) = torque, oz-in
- \( \omega \) = angular velocity, radians/sec
- \( v \) = linear velocity, in/sec
- \( L \) = length, inches
- \( R \) = radius, inches
- \( \rho \) = density, ounces/in³
- \( g \) = gravity constant, 386 in/sec²

The formula for load inertia converts linear inertia into the rotational equivalent as reflected to the motor shaft by the leadscrew.

Problem

Find the torque required to accelerate a 200-lb steel load sliding on a steel table to 2 inches per second in 100 milliseconds using a 5 thread/inch steel leadscrew 36 inches long and 1.5 inches in diameter. Assume that the leadscrew has an Acme thread and uses a plastic nut. Motor inertia is given as 6.56 oz-in². In this example, we assume a horizontally oriented leadscrew where the force of gravity is perpendicular to the direction of motion. In non-horizontal orientations, leadscrews will transmit varying degrees of influence from gravity to the motor, depending on the angle of inclination. Compumotor Sizing Software automatically calculates these torques using vector analysis.

1. Calculate the torque required to overcome friction. The coefficient of static friction for steel-to-steel lubricant contact is 0.15. The median value of efficiency for an Acme thread and plastic nut is 0.65. Therefore:

\[
F = \mu_s W = 0.15 (200 lb) = 480 \text{ oz}
\]

\[
T_{\text{Friction}} = \frac{F}{2\pi p e} = \frac{480}{2\pi \cdot 5 \cdot 0.65} = 23.51 \text{ oz-in}
\]

2. Compute the rotational inertia of the load and the rotational inertia of the leadscrew:

\[
J_{\text{Load}} = \frac{W}{2\pi p} = \frac{200 \text{ lb}}{2\pi \cdot 5 \text{ rev}} = 16 \text{ oz-in}^2
\]

\[
J_{\text{Leadscrew}} = \frac{\pi L p R^4}{2} = \frac{\pi}{2} (36 \text{ in}) (4.48 \text{ oz}) (0.75 \text{ in})^4 = 80.16 \text{ oz-in}^2
\]

3. The torque required to accelerate the load may now be computed since the motor inertia was given:

\[
T_{\text{Acceleration}} = \frac{1}{g} \left( J_{\text{Load}} + J_{\text{Leadscrew}} + J_{\text{Motor}} \right) \frac{\omega t}{\omega}
\]

\[
\omega = 2\pi \left( \frac{5 \text{ in}}{0.01 \text{ sec}} \right) = \frac{20\pi}{0.1 \text{ sec}}
\]

\[
= 386 \text{ in/sec}^2 (4.99 + 80.16 + 6.56(\text{oz-in}^2)) \cdot \frac{20\pi}{0.1 \text{ sec}} = 149 \text{ oz-in}
\]

\[
T_{\text{Total}} = T_{\text{Friction}} + T_{\text{Acceleration}}
\]

\[
T_{\text{Total}} = 23.51 \text{ oz-in} + 149 \text{ oz-in} = 172.51 \text{ oz-in}
\]